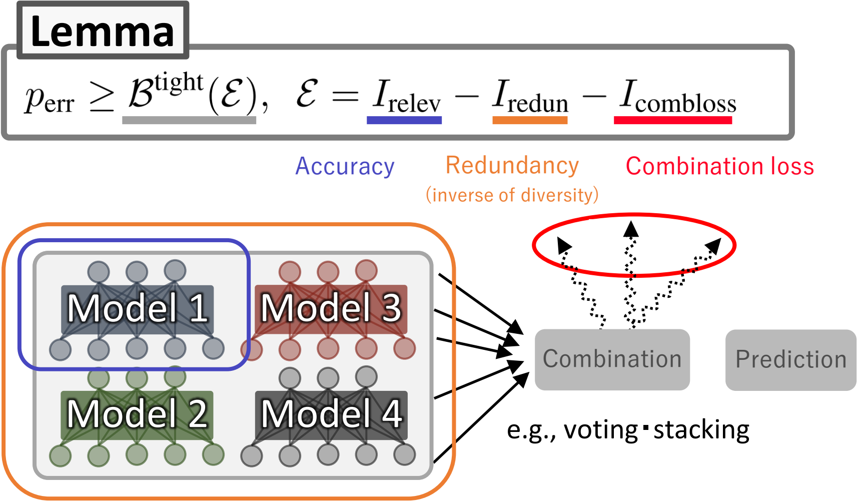

Lemma 3.1 (Decomposition of error rate lower bound into three metrics). *Let* $\mathcal{U}(p) = \mathcal{H}_2(p) + p\log_2(Y_{\max} - 1)$ *and* $\mathcal{U}'(p) = \frac{d\mathcal{U}}{dp}(p)$, *and let* $p_0 \in [0, 1]$ *be the approximate error rate. Then, for any* p_0, *the error rate* p_{err} *is bounded as*

$$p_{\text{err}} \geq \mathcal{B}^{\text{tight}}_{p_0}(\mathcal{E}(\mathbf{O}, Y, \hat{Y}))$$

$$:= p_0 + \frac{\mathcal{U}'(p_0)}{4} \left\{ 1 - \sqrt{1 - 8\frac{H(Y) - \mathcal{E}(\mathbf{O}, Y, \hat{Y}) - \mathcal{U}(p_0)}{\mathcal{U}'(p_0)^2}} \right\}$$